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Warped Kaluza-Klein dark matter

Andrew R. Frey, Rebecca J. Danos and James M. Cline

Department of Physics, McGill University, Montréal, QC, H3A 2T8, Canada
E-mail: frey@physics.mcgill.ca, rjdanos@physics.mcgill.ca, jcline@physics.mcgill.ca

Abstract: Warped compactifications of type IIB string theory contain natural dark matter candidates: Kaluza-Klein modes along approximate isometry directions of long warped throats. These isometries are broken by the full compactification, including moduli stabilization; we present a thorough survey of Kaluza-Klein mode decay rates into light supergravity modes and Standard Model particles. We find that these dark matter candidates typically have lifetimes longer than the age of the universe. Interestingly, some choices for embedding the Standard Model in the compactification lead to decay rates large enough to be observed, so this dark matter sector may provide constraints on the parameter space of the compactification.

Keywords: Flux compactifications, Cosmology of Theories beyond the SM

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Dedicated to the memory
of Lev Kofman

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Contents

1 Introduction

The search for connections between string theory and observation is a longstanding one since such a link would provide a large hint about the ultraviolet completion of our effective field theory, the ultimate description of physics beyond the Standard Model (SM). Recent and ongoing attempts include constructing string models of inflation (see [1] for a recent review and more references) or SM extensions (see [2] for one example of the Minimal Supersymmetric SM and [3] for an example of a Grand Unified Theory). In this paper, we revisit the argument that dark matter (DM) may have an intrinsically extra dimensional nature in many compactifications of string theory. We will show that the best-studied compactifications contain a naturally long-lived DM candidate particle. Furthermore, we argue that the decay of the DM candidate into SM particles can constrain some embeddings of the SM within the compactification.

Our candidate DM particle is a Kaluza-Klein (KK) mode of the compactification. KK modes have been considered as DM candidates in many contexts [4–17] (see [18, 19] for an alternative stringy origin of DM). We focus on a specific corner of the landscape of 4D solutions of string theory, warped flux compactifications of type IIB string theory on Calabi-Yau manifolds. These compactifications contain warped Klebanov-Strassler throat regions [20] which generate mass hierarchies *à la* Randall-Sundrum [21]. Since the throats are six-dimensional, they can have approximate isometries along the five angular directions, the Einstein-Sasaki manifold $T^{1,1}$, with a corresponding charge that is almost conserved. Charged KK modes can therefore be long-lived cosmologically, as was pointed out by [22]. Subsequently, [23, 24] argued that such states at TeV mass, like thermal WIMPs, can naturally have the correct relic density for DM candidates, although they only considered KK modes of the graviton, rather than the whole supergravity spectrum.

Ref. [25] was the first to try to identify a specific angular KK DM candidate in the Klebanov-Strassler throat geometry, in terms of the 10D supergravity spectrum of states and its dimensional reduction to 5D [26]. In this paper, we conduct a more exhaustive study of decay rates for the lightest charged bosonic KK modes, making order of magnitude estimates, extending the work of [25] and improving it using a better understanding of warped compactifications. We demonstrate that decays to light supergravity fields, including gravitons and light stabilized moduli, are all slow compared to the age of the universe for TeV scale DM. We also consider decays to SM particles in different brane models for the SM, a Randall-Sundrum–like D3-brane model and a D7-brane model. Finally, we discuss decays involving tunneling to different regions of the compactification, along the lines of ref. [23, 27].

One of our most striking observations is that D3-brane SM sectors can naturally yield DM decay rates in an observable range. Assuming that the relic density estimates of [23, 24] apply, DM decay therefore gives direct constraints on the parameter space of this region of the string landscape. Since finding such constraints is a rare opportunity, these models deserve further detailed study. We also find that D7-brane models can yield observable decay rates, although this statement is more model-dependent than in the D3 case. We emphasize that we did not specifically seek to build models with observably large decay rates (in contrast to [28]), but rather we searched for natural consequences of common embeddings of the SM in this class of string compactification.

The plan of the paper is as follows. In the next section, we give a brief review of the class of compactifications we study, including the warped throat regions of interest. In section 3 we review and synthesize the spectrum of these compactifications, including KK modes, light supergravity modes, and SM particles on D-branes. Section 4 discusses the interactions of our DM candidate with intermediate states and all the most likely decay products. We use these interactions to estimate the partial decay rates in section 5. In section 6, we summarize our results, compare them to the previous literature on KK modes as DM candidates in string compactifications, and list directions of interest for future research.

2 Review of compactification and warped throat

The best-understood warped compactifications of string theory are conformally Calabi-Yau (CY), as described by [29–31] (see [32] for a review). In particular, the 10D metric takes the form

$$ds^2 = e^{2A}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A}d\tilde{s}^2 \tag{2.1}$$

for CY metric $d\tilde{s}^2$ and warp factor A that depends on the internal space.[1] These compactifications develop "throat" regions where the warp factor becomes small and which can act as approximate Randall-Sundrum two-brane compactifications [21, 31, 33, 34]. The warp factor is sourced by 3-form field strengths of the 10D supergravity; these fluxes also stabilize a subset of the moduli. The remaining moduli can be stabilized either by nonperturbative physics or higher-derivative operators [35–37]. After complete moduli stabilization, there should be a small positive cosmological constant and deformations of the compact manifold, but we ignore these for the most part.

The simplest warped throats in these compactifications were first described by Klebanov and Strassler (KS) [20] in the context of gauge/string theory dualities. KS throats are well-approximated by a warped conifold geometry [38, 39] away from both their tip and the region that they join to a compactification. In this regime, the warp factor is such that the Minkowski dimensions and radial direction of the conifold form anti-de Sitter spacetime; the 10D spacetime factorizes into $AdS_5 \times T^{1,1}$, where $T^{1,1}$ is the 5D base of the conifold (and is topologically $S^2 \times S^3$). It is important to note that this approximate geometry is dual to a supersymmetric conformal field theory as described in [38].

We work in coordinates such that the AdS_5 metric is

$$ds^2 = e^{-2kz}\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\ . \tag{2.2}$$

With this radial coordinate, the CY metric in the throat takes the form

$$d\tilde{s}^2 = e^{-2kz}\left[dz^2 + \frac{1}{k^2}d\hat{s}^2\right]\ , \tag{2.3}$$

[1] For convenience, μ indices are denoted as raised with the Minkowski metric and warp factors are counted explicitly.

where $d\hat{s}^2$ is the metric of $T^{1,1}$. The throat runs from $z = 0$, where it glues to the compact bulk, to the tip at z_0, and the warp factor at the tip is $w \equiv e^{-kz_0}$. At the tip of the throat, the KS geometry is $\mathbb{R}^{1,3} \times \mathbb{R}^3 \times S^3$ with a smooth transition to $AdS_5 \times T^{1,1}$. For the most part, we can simply replace the smooth tip region by cutting off the geometry at z_0. In the UV, at $z \approx 0$, the warp factor approaches a constant and $d\tilde{s}^2$ approaches the bulk CY metric smoothly. As we review below, the KK scale is wk; since we are interested in KK dark matter, we generally take $wk \sim$ TeV.

The $T^{1,1}$ factor has an $\mathrm{SU}(2) \times \mathrm{SU}(2)/\mathrm{U}(1)$ isometry group, which is preserved by the full KS solution in the noncompact limit. This isometry and the associated mode decomposition on the $T^{1,1}$ have been discussed in [26]. States are labeled by each of their two SU(2) total spins j and l and by $r = (j_3 - l_3)/2$; due to the U(1) quotient, states with the same j, l but different r are nondegenerate. We discuss the charges and masses of KK states and other fluctuations in 3 below. The isometry must be broken when the throat is glued to a compact Calabi-Yau manifold (as those have no isometries), as we describe in 4.1.1.

A note about our conventions: throughout, we work in the 4D Einstein frame appropriate for a stabilized compactification volume. In this frame, the 4D and 10D gravitational scales are related by

$$M_p^2 \approx M_s^8 V_w \,, \quad V_w \equiv \int d^6y \sqrt{\tilde{g}} e^{-4A} \,, \tag{2.4}$$

where A is the background warp factor and the approximation symbol indicates that we have redefined the string scale by appropriate factors of 2π and the (nearly unity) string coupling. The relation (2.4) is derived from the usual Einstein-Hilbert term for the unwarped 4D metric; the determinant of the 10D metric contains a factor e^{-2A} when written in terms of the spacetime metric and $\tilde{g}$, and there is an additional factor of e^{-2A} in the contraction of the 4D Ricci tensor. Note that masses are not rescaled in transforming from the 10D frame to this Einstein frame. Also note that the warped volume V_w includes the bulk of the CY and is not necessarily dominated by the volume of the KS throat; further, we can choose coordinates such that $A \sim 0$ and $\tilde{g}_{mn} \sim 1$ (for coordinates of dimension length) in the bulk.

3 Spectrum of fluctuations

We can now consider the mass spectrum of excitations of the warped compactification, focusing on the behavior in the throat region. We are particularly interested in the approximately conserved charge of each mode. While most of this section reviews a range of results from a diverse literature, we believe that the synthesis and interpretation is novel.

We start with a discussion of KK modes in the KS throat, continue with light supergravity modes (that is, fields with mass less than the warped KK scale wk), and finally discuss fields on branes (which represent SM degrees of freedom). A summary of the various states we consider is presented in table 1.

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Particle		definition	throat wave function
γ ($\delta\Gamma$)	$T^{1,1}$ breathing mode	eq. (3.2)	$k^3 M_s^{-4}\, w^{1+\nu}\, e^{(2+\nu)kz}$
$\gamma^\star$ ($\delta\Gamma^\star$)	angular excitation of γ	eq. (3.2)	$k^3 M_s^{-4}\, w^{1+\nu^\star}\, e^{(2+\nu^\star)kz}\, Y(\theta_i)$
c	universal volume modulus	eq. (3.4)	constant
a	universal axion	eq. (3.5)	$k^{-3} M_p^{-1} e^{-4kz}$
$a^\star$	angular KK axion	section 3.2	$k^{-3} M_p^{-1} [e^{-4kz}$ or constant$] \times Y(\theta_i)$
ϕ, Θ_3	D3-brane fluctuation + superpartner	eq. (3.7) eq. (3.8)	NA
χ, Θ_7	D7-brane fluctuation + superpartner	eq. (3.10) section 3.3.2, 4.2.2	constant
Background deformation		definition	throat wave function
$\Delta\Gamma$	$T^{1,1}$ breathing mode	eq. (4.1)	$w^4\, e^{2kz}\, Y(\theta_i)$

Table 1. The relevant fluctuations of light mass particles and of the warped background. In all cases, $Y(\theta_i)$ stands for $Y_{(1,0,0)}(\theta_i)$ or $Y_{(0,1,0)}(\theta_i)$

3.1 KK modes

In this section, we review the wave functions and spectra of the lightest KK modes, particularly looking for the lightest mode with angular charge. Assuming a thermal history for the KK modes (as in [23]), angular charge should accumulate in this state (with some caveats, see 3.2 below), so it is our DM candidate.

In general, the lightest KK modes are localized in the longest KS throat, at least when the warped KK scale is less than the curvature scale of the bulk CY (see, for example, [23, 40–42]).[2] Therefore, we only need to be concerned with the form of KK modes in the throat, and we can proceed by dimensionally reducing on $T^{1,1}$ and then AdS_5 with boundary conditions at $z = 0$ and z_0. The boundary conditions at $z = z_0$ control the mass, so the KK scale for localized modes is set by wk, though the 4D masses of the KK modes are largely insensitive to the form of the boundary conditions [25].

We thus need to know the contributions to the 5D mass of various fields. These include contributions both from the KK reduction on $T^{1,1}$ and from the moduli stabilization. The angular motion on $T^{1,1}$ gives contributions of order $\Delta(m_5^2) \sim k^2$ (derived in detail in [26]); further, motion on the "S^2 factor" of $T^{1,1}$ generates a similar mass due to a centripetal barrier near the tip of the throat [22, 41]. Moduli stabilization includes a classical component due to 3-form flux, which may be as large as $\Delta(m_5^2) \sim k^2$ or much smaller, depending on the interaction of the 10D field with the flux (see [42] for example). Nonperturbative or α' corrections to the compactification can also generate a mass; we assume that these are small compared to the AdS_5 scale k in order to assure that these corrections do not deform the geometry too much.

Note that, even though the precise KK spectroscopy of the throat is known [26], the identity of the lightest charged KK mode is highly model-dependent (note that the discussion of [25] only considered geometric KK reduction). In practice, then, we choose a charged mode as a proxy for the true lightest charged state. For the purposes of order-of-magnitude

[2] In any event, delocalized KK modes would not preserve angular charge because the bulk CY does not respect the throat's isometries.

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estimation, this should be reliable as long as the dominant decay mechanism is the same, and we elaborate guidelines for converting our decay rates to alternate possibilities.

Fortuitously, the simplest mode to consider is also the mode identified as the lightest charged state using purely geometric KK reduction. The lightest states are the breathing mode of the $T^{1,1}$ (and an associated fluctuation in the warp factor which we can ignore for order-of-magnitude calculations) and an axion partner, both with $(j, l, r) = (1, 0, 0)$ or $(0, 1, 0)$ [25]. We can parameterize the breathing mode by deforming the CY metric $d\tilde{s}^2$ as

$$d\tilde{s}^2 = e^{-2kz}\left[dz^2 + \frac{e^{2\Gamma(z,\theta)}}{k^2}d\hat{s}^2\right] . \tag{3.1}$$

The charged KK modes are small fluctuations of Γ, which we denote $\delta\Gamma^\star$ (the $\star$ representing angular excitation). These fields saturate the Breitenlohner-Freedman bound [43] in AdS_5, and the 4D mass is of order wk, the warped KK scale. Γ also has KK modes with no angular motion, which are massless in 5D. Those fluctuations are actually part of the universal volume modulus, but they also contain radial KK excitations denoted $\delta\Gamma$ with 4D mass of approximately wk (somewhat larger than the mass of $\delta\Gamma^\star$). These uncharged KK modes are important as intermediate states in decay amplitudes of $\delta\Gamma^\star$.

We also need the properly normalized wave function for the KK modes on the compactification in order to write interactions in terms of 4D fields, which we denote with lower case γ. Massive KK modes are localized in the KS throat, predominantly near the tip. In fact, the wave functions of the lightest radial KK modes are given by growing exponentials in z to a reasonable approximation [25], which are sufficient for our estimates. For example, a breathing mode fluctuation with angular momentum (j, l, r) has an approximate wave function

$$\begin{aligned}\delta\Gamma_{(j,l,r)}(x, z, \theta) &\approx \frac{\gamma_{(j,l,r)}(x)}{M_p}\left(\frac{V_w k^6}{V_{T^{1,1}}}\right)^{1/2} w^{1+\nu}Y_{(j,l,r)}(\theta)e^{(2+\nu)kz} \\ &\approx \frac{k^3}{M_s^4}\gamma_{(j,l,r)}(x)w^{1+\nu}Y_{(j,l,r)}(\theta)e^{(2+\nu)kz}\end{aligned} \tag{3.2}$$

within the throat, where $\nu^2 = 4+m_5^2/k^2 \sim 1$ for 5D mass m_5 and the factor of M_p is needed for canonical normalization of the 4D field γ.[3] $V_{T^{1,1}}$ is the angular volume, an order unity constant, which we henceforth drop. Considering only the angular dimensional reduction, we would obtain $\nu^\star \equiv \nu(\delta\Gamma^\star) = 0$ and $\nu \equiv \nu(\delta\Gamma) = 2$; however these values are increased by flux contributions. Given the spacing of geometric 5D masses, it seems unlikely that $\nu^\star$ is much larger than 2 for the true lightest charged state. In addition, the wave function dies off exponentially quickly outside the throat, so we can treat it as zero in that region. The normalization is fixed up to order 1 factors by the condition that

$$\frac{1}{V_w}\int d^6y\sqrt{\tilde{g}}\, e^{-4A}\delta\tilde{g}_{mn}\delta\tilde{g}^{\widetilde{mn}} = \frac{\gamma^2}{M_p^2}\, , \tag{3.3}$$

[3]The true radial wave function is a product of an exponential and a sum of Bessel functions, but [25] showed that the pure exponential behavior is a good approximation for the lowest modes in the radial KK tower, at least for order-of-magnitude calculations.

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as given in [44]. The wave function (3.2) of course ignores the fact that some angular modes must vanish at the tip of the throat [22, 41], but we can account for that effect by hand.

3.2 Moduli and other light supergravity modes

The moduli and light fields of the compactification have been a topic of some interest in the literature, due to their relevance for the effective field theory of warped compactifications (see, for example, [32, 42, 44–48]). Our main interest in light fields will be as decay products of our DM candidates (we assume that massive moduli can decay rapidly to SM particles); they may also be obstructions to the cosmological population of the KK DM candidates, but we leave that possibility for future studies. In this discussion we consider light states to be all states with 4D mass significantly less than the warped AdS_5 scale wk. These are states that are massless before moduli stabilization effects. As long as their masses remain light, their wave functions are expected to be similar to what they would be without moduli stabilization [42]; semiclassically, when their mass reaches $\sim wk$, it is energetically favorable for their wave functions to localize near the tip of the throat — but it is not at a lower mass.

To identify the light modes, we start by asking what states might be massless if we ignore moduli stabilization effects. Since the wave functions of light modes typically spread through the entire compactification, they cannot be reliably studied just by considering dimensional reduction in the throat in a simple way. Specifically, from the point of view of the AdS_5 throat, the boundary conditions at $z = 0$ mix many different fields, which leads to surprising behavior. Nonetheless, the approximate isometry of the throat ensures that we can break down fields by angular charge sectors in the throat. Furthermore, even though the radial wave functions may not take the form suggested by pure AdS_5 reduction, all possible cases of light fields are represented by fields with vanishing 5D mass as given by reduction on $T^{1,1}$. We can proceed by examining angular charge sectors one at a time.

The most familiar sector is likely the uncharged one, which has wave functions uniform on the $T^{1,1}$. For instance, all warped CY compactifications have an overall volume modulus and an associated axion (sometimes called the universal volume modulus and universal axion respectively). In addition, a conifold throat has two moduli associated with the two ways to resolve the singular conifold point; these are identified as Betti forms in [26]. For the KS throat, the modulus associated with the small resolution of the conifold is lifted by topology (essentially it is incompatible with IR boundary conditions), so it has only higher KK modes. The modulus associated with the deformation of the conifold is stabilized by the 3-form flux on the throat. While this could still be a light mode if the compactification is large, we focus on the universal volume modulus and axion due to the relative simplicity of their profiles in the extra dimensions. There may, in fact, be other uncharged moduli, depending on the CY manifold, but we take the universal volume modulus and axion to be representative of them as well.

By now, the universal volume modulus is well-understood [46, 48]. However, it is not described by an overall rescaling of the CY metric $d\tilde{s}^2$ accompanied by an appropriate fluctuation of the warp factor; rather, it is most simply described by a shift of the warp factor

$$e^{-4A} \to e^{-4A} + \frac{c(x)}{M_p} \tag{3.4}$$

along with metric fluctuations that depend on the spacetime derivatives of c, the volume modulus (given here with canonical normalization up to factors of order unity). In particular, note that $c(x)$ is independent of the compact dimensions.

Similarly, the universal axion is a fluctuation of the 4-form potential based on the Kähler form of the CY and can be written with all four legs in the compact dimensions [48]. This involves multiple components of the 4-form when written in a given set of coordinates, and all these components are required to produce a single massless 4D field (on a CY with multiple Kähler moduli, some of these components could be independent fields, but not if there is only the universal volume modulus). These modes are also accompanied by fluctuations of other supergravity field strengths. However, we are only making rough order-of-magnitude calculations, and we have checked that these effects are unimportant for our purposes. Ignoring those other field strengths, it is a simple calculation to see that the 4-form scales like e^{-4kz} in the throat and like a constant in the bulk [48]. Picking a sample component, we have

$$C_{z\theta\phi\psi} \approx e^{-4kz}\frac{a(x)}{k^3 M_p} \text{ (throat) or } \approx \frac{a(x)}{k^3 M_p} \text{ (bulk)}\,, \tag{3.5}$$

where we define the scale of the angular directions as k following (2.3) and use canonical normalization. The kinetic term is a 6D integral dominated by the bulk, which yields the normalization.

As discussed above, this modulus is stabilized by nonperturbative or α' effects, but its mass should be less than the warped KK scale to ensure that the corrections do not deform the geometry too strongly. For low mass, the wave function should be essentially the same as the classical wave function [42]. Note, though, that the wave functions are not constants in the throat, as suggested by modeling the throat as AdS_5 with simple boundary conditions. The reason is that the moduli are combinations of many 10D fields, which play off of each other in a complicated manner and are related by more complicated boundary conditions in the AdS_5 picture.

We now turn to the $(j, l, r) = (1, 0, 0)$ and $(0, 1, 0)$ charge sectors, which are identical in the $AdS_5 \times T^{1,1}$ geometry. Just considering the KK masses from the angular reduction, as discussed in [26] there are several massless states. These states include a 4D vector descending from the RR 4-form potential and scalars from the metric and 4-form potential taking the form of a Kähler modulus. Again, the AdS_5 dimensional reduction is not entirely reliable, but both of these types of massless states are possible on a general CY compactification. Either charged vectors or charged moduli can remove the angular charge during cosmological evolution. These would be the lightest charged states, so thermal equilibrium would eventually drive all the charge into the light states. For exactly massless states, this would remove the charge completely, as radiation redshifts more quickly than matter. However, even stabilized charged moduli present a problem. As long as the 4D mass of these is below the warped KK scale wk, their wave functions should be spread throughout the bulk of the compactification. Since the isometry of the throat is strongly broken in the bulk, these moduli are allowed to have interactions that violate angular charge conservation.

However, the zero modes of the massless vectors are projected out by O3-planes of string theory, so we can restrict to compactifications with O3-planes. Alternately, the particular angular harmonic associated with the massless vector may not be associated with a massless mode on the bulk CY. In either case, the boundary conditions at the UV end of the AdS_5 throat are incompatible with the zero mode, so only higher radial KK modes are allowed. Analyzing the cosmological transfer of angular charge into these vectors is an interesting question for future studies of the detailed cosmology. In this paper, however, we simply assume that these vectors are projected out by the bulk CY physics (possibly an orientifold plane).

The light scalars in this charge sector take the form of Kähler moduli of the CY manifold, a metric scalar and a 4-form axion. It is well-known that the number of Kähler moduli is controlled by topological invariants of the CY; if there is only one Kähler modulus, these zero modes must be projected out, as discussed for the vector above. On the other hand, if the CY has multiple Kähler moduli, it is possible that these states can remain massless at the classical level. Furthermore, the 3-form flux does not stabilize these moduli. These moduli are stabilized either by α' corrections or nonperturbative effects. Again, if this mass is less than the warped KK scale wk, these moduli could provide a cosmological sink for the charge. While we leave a discussion of this possibility for future work, we estimate the decay rate of charged KK modes into these "charged" moduli on the chance that their scattering rates are too weak to permit thermalization.

In order to calculate decay rates, we need the wave functions of these moduli. Treating them as 5D fields and dimensionally reducing on AdS_5 indicates that these modes should have constant profiles in the KS throat. However, it is possible that, like the universal volume modulus and axion, the true zero modes are a mixture of multiple components, which can change the radial behavior. An additional complication is that the appropriate tensor angular harmonics on $T^{1,1}$ are unknown. Therefore, we use axions (potential decay products themselves) also as proxies for moduli. For the axion, we consider both a constant wave function and one that scales like e^{-4kz}, like the universal axion. In all cases, the wave functions should be approximately constant in the bulk (because they are given by harmonic forms).

There is a final possibility for light scalars. The $(j, l, r) = (1, 1, \pm 2)$ charge sectors also have massless scalars after reduction in the angular directions. Assuming that these modes are massless after matching to the bulk CY metric, they are pairs of metric scalars, as expected for complex structure moduli. Again, we should not take the AdS_5 dimensional reduction literally, but we can also consider the case of charged complex structure moduli. These moduli are generically stabilized by 3-form flux, like the deformation modulus of the conifold. Like the deformation modulus, these may be heavy or light fields, but we consider other moduli as proxies for them, since we are just working to order of magnitude.

We conclude this section by commenting on effects of moduli stabilization on wave functions in the throat. As mentioned earlier, if the mass scale of moduli stabilization is below the warped scale wk, the wave function should not be much distorted from the massless case. However, at masses more than wk, the mode should accumulate near the bottom of the longed throat, behaving essentially like an excited KK mode [42]. Therefore, in those cases, we can simply use our results without including light supergravity states. There is one caveat, however. For moduli stabilized by quantum mechanical or α' corrections, such a high mass may or may not indicate that the throat has been deformed strongly by the physics that stabilize the moduli, so it is possible that the geometry is not a small perturbation of the KS background, breaking our assumptions (specifically, when we discuss deformations of the throat later, we assume that those deformations are small). This may be an important concern for high-scale inflation models; however, our results should be parametrically similar as long as the throat remains approximately isometric.

3.3 Brane modes

In the context of IIB string theory compactifications, the Standard Model (SM) is typically understood as being supported on a D-brane or stack of D-branes, either D3-branes or D7-branes in warped CY compactifications. We are interested in the case in which the SM branes are located in the same KS throat as the KK modes of interest; this is somewhat natural due to the TeV scale of the throat. It is also possible that the branes are located in the bulk (for D7-branes) or another throat. In that case, any warp factor quoted in this subsection or 4.2 should be replaced by that for the appropriate throat.

3.3.1 D3-brane Standard Model

The brane scalars correspond to transverse motion of the D-branes; these are the SM Higgs fields or scalar superpartners of the SM fermions (either of which can decay quickly to SM fermions). The kinetic terms just follow from the DBI action on the branes; in the case of D3-branes, it is

$$\mathcal{L} = -\frac{1}{2}\frac{\mu_3}{g_s}\mathrm{tr}\left(\tilde{g}_{mn}\partial_\mu\phi^m\partial^\mu\phi^n\right) \ . \tag{3.6}$$

The brane scalars are just the coordinates of the brane on the conifold and the trace is over the non-Abelian degrees of freedom on the branes. We consider a single angular coordinate (along an equator of the S^3 at the tip) and reduce to a single diagonal component of the gauge matrix. With metric (2.3), we find

$$\mathcal{L} \approx -\frac{1}{2}\frac{\mu_3}{g_s}\frac{w^2}{k^2}\left(\partial\phi\right)^2 \ , \tag{3.7}$$

so the canonically normalized scalars are rescaled by $\sqrt{\mu_3/g_s}(w/k)$. (Alternately, we can change to Riemann normal coordinates at the brane position, in which case there is no need to rescale by w/k.)

To see whether the brane scalars give a representative decay rate, we also check for decays into D3-brane fermions. From [49] (see also [50, 51]), the fermion kinetic term on a D3-brane at z_0 is

$$\mathcal{L} = \mu_3 w^4\left[-\frac{1}{2}w^{-1}\bar{\Theta}_3\not{\partial}\Theta_3\right] \ , \tag{3.8}$$

where μ_3 is the brane charge and Θ_3 is a 10D fermion that contains all the 4D fermionic degrees of freedom. In the sequel, we use Θ_3 to represent one 4D fermion. Up to factors of order unity, then, the canonically normalized fermions are $\sqrt{\mu_3} w^{3/2}\Theta_3$.

Finally, there are worldvolume gauge fields (the SM gauge fields) on the D3-branes. We do not consider decays into the gauge fields, however.

3.3.2 D7-brane Standard Model

We now consider the case in which the SM lives on D7-branes extended in the throat. Our results are largely insensitive to the embedding of the D7-brane in the compactification, but we show how the D7-brane kinetic term (3.10) matches that for a specific embedding in appendix A. We assume that the D7-branes do not extend to the bottom of the throat (that is, the branes extend to a maximum radial coordinate of $z_1 \ll z_0$, where $w_1 = e^{-kz_1}$). Furthermore, like our treatment of the throat, we replace the smooth IR tip of the brane with boundary conditions at z_1. Except near the tip of the brane, z lies along the brane, so the transverse coordinates lie in the $T^{1,1}$.

In the case of the D7-brane, there is a single complex scalar field χ, corresponding to fluctuations around the static embedding. The kinetic term of the brane scalars is controlled by the metric pulled back to the fluctuating brane. Additionally, the light scalars on a D7-brane have a constant profile in the compact dimensions of the brane (which agrees with [52] for D7-branes in a KS throat). These are the appropriate final states to consider for decays of the dark matter candidates: excited KK modes of the D7-brane are more massive than the gravitational KK modes we consider (D7-brane KK mode masses are set by the warped scale at z_1 suppressed by flux quantum numbers, and this suppression is insufficient to reduce the mass to the warped KK scale at z_0).

It is true that the light scalar χ must in fact be stabilized, although we treat it as massless. First, the mass scale of χ may be much less than the warped scale at the bottom of the D7-brane $w_1 k$; indeed, if χ is stabilized by supersymmetry breaking, this situation may be natural. In that case, the χ wave function is approximately constant, following the same logic as for moduli. Furthermore, we want to estimate the interaction of the KK modes with SM particles, which are very low mass compared to $w_1 k$. Therefore, we are justified in treating the χ particles as massless.

For brane scalars that depend only on the external spacetime $\chi = \chi(x^\mu)$, we can approximate the pulled back metric as

$$ds_8^2 \approx \left(e^{2A}\eta_{\mu\nu} + 2e^{-2A}\tilde{g}_{\chi\bar{\chi}}\partial_{(\mu}\chi\partial_{\nu)}\bar{\chi}\right) dx^\mu dx^\nu + e^{-2A}\tilde{g}_{ij}dy^i dy^j \,, \tag{3.9}$$

where y^i are the four compact coordinates along the brane. There are generically off-diagonal terms of the form $\partial_\mu\chi dx^\mu dy^i$, but these lead to parametrically similar contributions to the kinetic term (which we see for a specific example in appendix A). We choose coordinates in the stabilized compactification such that χ matches from the bulk to the throat, and we treat $\tilde{g}_{\chi\bar{\chi}}$ as approximately constant and order unity in the bulk. Then the approximate kinetic term from the DBI action is

$$\mathcal{L}_\chi \approx -\frac{\mu_7}{g_s}\int d^4y\sqrt{\tilde{g}_4}\, e^{-4A}\tilde{g}_{\chi\bar{\chi}}|\partial\chi|^2 \,. \tag{3.10}$$

In the throat, $\tilde{g}_{mn} \propto e^{-2kz}$, so the integrand dies off farther into the throat. That means the kinetic term is dominated by the part of the D7-brane that extends into the bulk CY manifold. Therefore,

$$\mathcal{L}_\chi \approx -\frac{\mu_7}{g_s} V_{4w} |\partial\chi|^2 \,, \tag{3.11}$$

where V_{4w} is the warped 4-volume along the brane. In the bulk, where the warp factor is $A \sim 0$, we can approximate it by $V_{4w} \sim V_w^{2/3} = (M_p^2/M_s^8)^{2/3}$. This gives us immediately the canonical normalization for χ.

Similarly to our treatment of the D3-brane Standard Model, we do not consider the brane gauge fields (which include 4D scalars for a D7-brane). We do, however, consider the D7-brane fermions, estimating their interactions based on those of the brane scalars, as explained below.

3.4 Supergravity fermion KK modes

The reader may note that we have not discussed modulini or KK modes of the supergravity fermions, despite the fact that the lightest charged state might very well be fermionic. The reason is two-fold. First, due to the fact that the simplified background we consider is supersymmetric, we expect that the decay rates of fermionic states should be similar to those of bosonic KK modes. Second, the calculation of fermionic decay rates is technically more difficult, as we now explain.

The difficulty lies in the simple fact that the decay products must always contain a fermion. Considering purely supergravity interactions is not difficult, although some care is needed in decomposing the 10D spinors into 4D spinors to find off-diagonal mixings and Yukawa couplings. While it is not as familiar as the bosonic action, the 10D supergravity is known for fermions (see [53, 54] and subsequent papers). However, while interactions between brane fermions and bosonic supergravity fields are known for D3- and D7-branes, mixings and mixed Yukawa terms between brane and supergravity fermions are not known in generality, particularly with warping included. To find these interactions, it is necessary to expand the κ-symmetric D-brane action to include more terms in the 10D superfields, as done in [55] for gravitino-brane fermion bilinears. While this is an interesting project, it is beyond the scope of the present work, and we leave it to the future.

4 Interactions

In this section, we describe the interactions of the KK modes, especially $\delta\Gamma^\star$ and $\delta\Gamma$, which we take to be representative of general angular KK modes in the KS throat, particularly the lightest one. We begin by discussing quadratic mixings and three-point interactions in the supergravity action and then move to interactions on branes. One or two other relevant interaction terms are also presented.

4.1 Supergravity interactions

All the interaction terms in the KS throat must be neutral under the approximate isometry, though branes that explicitly break the isometries can support charged interaction terms. As a result, deformations of the KS throat that break the approximate isometry, communicating the asymmetry of the bulk CY, is important to our analysis, so we discuss them first.

4.1.1 Isometry-breaking deformations

As we discussed above, compactification effects break the isometry of the KS throat, if for no other reason than that compact CY manifolds have no isometries. However, we demand that these be small perturbations to the throat geometry as the KS throat background would no longer be valid.

Since these deformations have nontrivial profiles on the $T^{1,1}$ in order to break the isometry, they are therefore also classified by the quantum numbers (j, l, r). The radial behavior of these deformations is governed by the conformal symmetry of the AdS_5 geometry (with small corrections in the full KS geometry), as has been discussed by [56]. In short, the radial behavior of the deformations is exponential in z with a coefficient determined completely by the angular quantum numbers. In fact, since the simplified geometry is dual to a conformal field theory, even α' and quantum corrections to the supergravity cannot change the radial behavior of the deformations![4] Generally speaking, because the KK wave functions are concentrated near the tip of the throat, the deformations that interact most with KK modes are those that decay least (or grow most) with increasing z.

Previous studies of throat KK modes have only considered decaying deformations because deformations that grow with z could eventually become $\mathcal{O}(1)$ and spoil the KS geometry. However, [57] has pointed out that growing deformations are allowed as long as their amplitude is sufficiently suppressed since the throat extends only a finite distance. They also identified such perturbations that would naturally be generated by nonperturbative effects and therefore have suppressed amplitudes (these deformations also preseve the supersymmetry of the background). In this paper, we study the decays of angular KK modes induced by these growing deformations; we find that they are the dominant decay modes despite the suppression of their amplitude. Section 6.1 compares our results to the previous literature.

As it happens, the growing deformations discussed in [57] are precisely the breathing mode of the $T^{1,1}$, which is accompanied by a fluctuation of the warp factor (which we have checked does not change the order-of-magnitude estimates we present later). There is also a similar deformation of the 4-form potential, but we focus on the metric breathing mode. This deformation is in fact forbidden in the tree level supergravity [31], so it must be sourced by nonperturbative or string physics, which suppresses its amplitude. For a small amplitude deformation $\Delta\Gamma$ of the breathing mode Γ, the deformation takes the form

$$\Delta\Gamma(z, \theta) \approx w^4 e^{2kz} Y_{(j,l,r)}(\theta) \tag{4.1}$$

for $(j, l, r) = (1, 0, 0)$ or $(0, 1, 0)$, where Y is the scalar angular harmonic function on $T^{1,1}$ [57]. Henceforth, we use a capital Δ to represent background deformations (as opposed to lower case δ for dynamical fluctuations).

[4] If we include the full KS throat background geometry, the subleading nonconformal terms in the behavior of the deformations may be affected by the α' and quantum effects, however.

In addition, ref. [57] discusses deformations of the form (3.1) with charge $(1/2, 1/2, \pm 1)$, which have radial form $\Delta\tilde{\Gamma} \sim w^4 e^{5kz/2}$. While we focus on the deformation (4.1), it is possible that the lightest charged state decays by neutralizing its charge against this second deformation. In that case, as we discuss in more depth in section 5.5, the decay rates may be enhanced by a factor of w^{-1}. However, this deformation can be forbidden by a discrete symmetry of the CY. In that case, if the lightest charged KK mode has charge $(1/2, 1/2, \pm 1)$, its decay rate could be suppressed compared to our estimates due to the need for insertions of multiple background deformations, for absorbing the angular charges. Since there are often multiple decay channels for the KK modes, this situation may or may not be important in practice.

4.1.2 Quadratic mixings

The background deformation $\Delta\Gamma$, since it carries angular charge, induces quadratic mixings between the charged and uncharged KK modes. While a careful accounting would include mixing in kinetic terms, we restrict our study to off-diagonal mass terms. From the 4D point of view, these couplings control the oscillation of $\delta\Gamma^\star$ into $\delta\Gamma$.

A brief note on philosophy: we are treating the background deformation $\Delta\Gamma$ as inducing a number of interactions, including quadratic terms, and we work in the interaction picture. This means we use the eigenmodes of the unperturbed background and introduce mixing diagrams when calculating decay amplitudes. An alternate approach, espoused by [24], is to diagonalize the quadratic action, including the new mixings, and then work out the interactions of the new eigenmodes. These approaches are equivalent, as long as the mixing is a small perturbation (otherwise one must resum the series of propagator insertions).

The appropriate potential is then [46]

$$\frac{\mathcal{L}}{M_p^2} = -\frac{1}{2V_w}\int d^6y\sqrt{\tilde{g}}\tilde{R} + \frac{g_s}{24V_w}\int d^6y\sqrt{\tilde{g}}\, e^{4A} G_{mnp}(\bar{G} - i\tilde{\star}_6\bar{G})^{\widetilde{mnp}}\,, \tag{4.2}$$

where $\tilde{g}$ and $\tilde{R}$ refer to the metric (3.1).[5] Both pieces of the potential contribute terms of the form $\Delta\Gamma\,\delta\Gamma\,\delta\Gamma^\star$; the first because the deformed fluctuating metric is no longer Ricci flat, and the second because the deformed fluctuating Hodge star $\tilde{\star}_6$ scales differently for different numbers of angular legs on $\bar{G}_3$. Specifically, the 6D Ricci scalar of (3.1) is

$$\tilde{R} = e^{2kz}\left[20k^2\left(e^{-2\Gamma}-1\right)+50k\partial_z\Gamma - 5\partial_z^2\Gamma - 30(\partial_z\Gamma)^2 - 8k^2e^{-2\Gamma}\hat{\nabla}^2\Gamma + 3k^2e^{-2\Gamma}\left(\hat{\nabla}\Gamma\right)^{\hat{2}}\right], \tag{4.3}$$

from which we can take the appropriate terms by defining $\Gamma = \Delta\Gamma + \delta\Gamma + \delta\Gamma^\star$. To find the contribution from the flux, we note that $\bar{G} = i\tilde{\star}_6\bar{G}$ on the conifold, so

$$(\bar{G} - i\tilde{\star}_6\bar{G})^{\widetilde{z\theta\phi}} = k^4e^{6kz}e^{-4\Gamma}\left(1-e^{-\Gamma}\right)\bar{G}^{z\hat{\theta}\hat{\phi}}\,,\quad (\bar{G} - i\tilde{\star}_6\bar{G})^{\widetilde{\theta\phi\psi}} = k^6e^{6kz}e^{-6\Gamma}\left(1-e^{\Gamma}\right)\bar{G}^{\hat{\theta}\hat{\phi}\hat{\psi}}\,. \tag{4.4}$$

[5] As mentioned above, we are ignoring terms that depend on the spacetime derivatives of the fluctuations since this does not affect the order of magnitude estimates. There are also other α' or nonperturbative corrections, but these also generically do not affect our estimates.

Following, for example, [42], we also note that the flux in the KS throat is approximately constant:

$$G_{z\theta\phi} \approx \frac{G}{\sqrt{6}k}\,, \quad G_{\theta\phi\psi} \approx \frac{G}{\sqrt{2}k^2}\,, \tag{4.5}$$

where G is a complex constant related to the number of RR and NSNS flux quanta.

Putting everything together, including the normalized wave functions (3.2), we find a mixing term between the charged and uncharged scalars. Since the wave functions are localized in the throat, the integration over y only runs over the throat and is dominated by the tip end. Furthermore, we find that the parameter ν cancels out. The mixing term is

$$\mathcal{L}_{\gamma\gamma^\star} \approx N k^2 w^4 \gamma(x)\gamma^\star(x)\,, \tag{4.6}$$

where N is a constant ranging from order unity to order 100, depending on the flux. Since the masses of γ and $\gamma^\star$ are of order wk, this induces a mixing angle of order w^2 between the Lagrangian fields and the mass eigenstates.

It is similarly possible to calculate mixing of $\delta\Gamma^\star$ with the universal volume modulus c of the compactification (which is described in [46, 48]). While there are derivative couplings, we focus on the coupling induced by the flux term in (4.2). Specifically, to first order in the modulus, the warp factor becomes $e^{4A} + e^{8A}c/M_p$; $\delta\Gamma^\star$ and $\Delta\Gamma$ each appear linearly through (4.4). For $\nu^\star < 4$, which we assume, the integral is dominated by the UV, so we find

$$\mathcal{L}_{\gamma^\star c} \approx \frac{M_s^4}{M_p k}\frac{w^{5+\nu^\star}}{4-\nu^\star}\gamma^\star c \tag{4.7}$$

for a canonically normalized modulus. We should note that mixing between KK modes and the modulus is not possible before the modulus is stabilized (due to orthogonality of the metric excitations); therefore, any mixing between the uncharged KK mode $\delta\Gamma$ and Kähler moduli must occur at least at quadratic order in the deformation $\Delta\Gamma$ due to the isometry. The mixing of (4.7) is linear in $\Delta\Gamma$, as we have seen; this is of course necessary to keep the angular integrals from vanishing.

4.1.3 Trilinear couplings

One type of decay we want to consider is the decay of our DM candidate to moduli of the compactification, since the moduli should be lighter than KK modes. Some of these decays exist in all the models we consider, so they give an upper bound on the lifetime of the KK dark matter. Here we collect the cubic interactions that we study.

We first consider cubic couplings with moduli, taking the universal volume modulus as a proxy for all the moduli. To start with, we note that any coupling linear in the KK modes and quadratic in moduli must involve at least one power of the deformation $\Delta\Gamma$. Any dimension-3 term of order $(\Delta\Gamma)^0$ would exist even in the absence of moduli stabilization, and, since the moduli can by definition take any expectation values in that case, such terms would generate tadpoles for the KK modes. Hence, $\delta\Gamma^\star$ has no dimension-3 couplings at lowest order to one uncharged and one charged modulus. Similarly, $\delta\Gamma$ has no zeroth order dimension-3 couplings to two uncharged or two charged moduli. This same argument applies, of course, to axions and to zero-derivative terms in the potential.

On the other hand, the deformation $\Delta\Gamma$ is sourced by the 10D effect that stabilizes the moduli. Therefore, there are dimension-3 interactions between, for example, $\delta\Gamma^\star$ and two universal volume moduli, which is suppressed by one power of $\Delta\Gamma$. This interaction also follows from the flux term of (4.2) and gives

$$\mathcal{L}_{\gamma^\star c^2} \approx \frac{M_s^4}{M_p^2 k}\frac{w^{5+\nu^\star}}{8-\nu^\star}\gamma^\star c^2 \tag{4.8}$$

for canonically normalized c.

We also consider dimension-5 cubic couplings, specifically those with three scalars and two derivatives. The simplest of these is actually a self-coupling of the volume modulus, which arises due to its nontrivial kinetic term (see [48]). In position and momentum space, it is

$$\mathcal{L}_{c^3} \approx \frac{1}{M_p}c(\partial c)^2 \approx -\frac{p_1\cdot p_2}{M_p}c^3\ . \tag{4.9}$$

There are also direct dimension-5 couplings between the KK modes and the moduli sector, which are easier to estimate for the axions. As an example, consider the coupling of $\delta\Gamma$ to two universal axions; the $T^{1,1}$ breathing mode appears in both the metric determinant and in the contraction of 4-form indices. If we focus on the $z\theta\phi\psi$ component of the universal axion, we can estimate this interaction as

$$\begin{aligned}\mathcal{L}_{\rm axion} &\approx M_s^8\int d^6y\sqrt{\tilde g}\, e^{4A}\delta\Gamma(x,y)\partial_\mu C_{z\theta\phi\psi}\partial^\mu C^{\widetilde{z\theta\phi\psi}}\\ &\approx \int_0^{z_0}dz\int d^5\theta\sqrt{\tilde g}\, k^6\delta\Gamma(x,y)e^{-2kz}\partial_\mu C_{z\theta\phi\psi}\partial^\mu C_z{}^{\widehat{\theta\phi\psi}}\ .\end{aligned} \tag{4.10}$$

In the first line of (4.10), the warp factors enter in the external and internal metric determinants, the external metric contraction, and the internal metric contractions; in the second line, additional exponentials of kz come from $\tilde g_{mn}$ in the determinant and contractions. Other z dependence is still implicit in the wavefunctions of the fluctuations. There are, of course, other components, but those give order one corrections to the interaction. Due to the localization of the KK mode near z_0, we can do the integral just over the throat; note that this leads to a suppression in the interaction because the axion wave function decays in the throat. In terms of canonically normalized $\gamma(x)$ and axions $a(x)$, the interaction term in momentum space is

$$\mathcal{L}_{\gamma a^2} \approx \frac{M_s^4}{M_p^2}\frac{p_1\cdot p_2}{(8-\nu)k^3}w^{1+\nu}\gamma a^2\ . \tag{4.11}$$

Here $p_{1,2}$ are the momenta of the two axions. We have assumed that $\nu<8$, which means the integral is dominated by the UV end of the throat; in the opposite case, take $w^{1+\nu}\to w^9$. Here, and in the following, we have replaced the warped volume with the string scale using (2.4).

There is also a direct cubic coupling between $\delta\Gamma^\star$ and two universal axions induced by $\Delta\Gamma$, which comes from the same kinetic term. We have the coupling

$$\mathcal{L}_{\gamma^\star a^2} \approx \frac{M_s^4}{M_p^2}\frac{p_1\cdot p_2}{(6-\nu^\star)k^3}w^{5+\nu^\star}\gamma^\star a^2\ , \tag{4.12}$$

assuming $\nu^\star < 6$ (which seems likely, as mentioned in section 3.1). If $\nu^\star > 6$, larger than our usual assumption, we take $w^{5+\nu^\star} \to w^{11}$.

In addition, we discussed in section 3.2 the possibility of moduli with angular quantum numbers in the throat. If these exist, our KK DM candidate can decay directly into one of these axions (plus a universal axion). We also mentioned two possibilities for the wave function of these charged axions, which we denote $a^\star$. The first is that the wave function decays as e^{-4kz} in the throat, like the universal axion's wave function. In that case, the interaction term takes the form of (4.11), with $\nu \to \nu^\star$, $\gamma \to \gamma^\star$, and $a^2 \to aa^\star$. The other extreme possibility, suggested by the AdS_5 dimensional reduction, is that the wave function of $a^\star$ is constant in the throat. In that case, we find

$$\mathcal{L}_{\gamma^\star aa^\star} \approx \frac{M_s^4}{M_p^2}\frac{p_1 \cdot p_2}{(4-\nu)k^3} w^{1+\nu^\star}\gamma^\star aa^\star \tag{4.13}$$

for $\nu^\star < 4$; take $w^{1+\nu^\star} \to w^5$ for $\nu^\star > 4$. Note that both of these possible wave functions give the same interaction for $\nu^\star \leq 4$.

We may further consider decays to spacetime gravitons $\delta\Gamma^\star \to h^2_{\mu\nu}$. However, [23, 24] showed using orthogonality of wave functions that this type of amplitude vanishes when $\delta\Gamma^\star$ is replaced by a KK graviton. We can generalize this result to show that no field can decay to gravitons. Suppose the 1PI effective action of our theory in Einstein frame takes the form $S = S_{EH} + S_m(g, \Phi)$, where Φ are all matter fields, S_{EH} is the usual Einstein-Hilbert action and that all backgrounds are covariantly constant. Then all the terms in the action that are first order in any fluctuation $\delta\Phi$ take the form

$$-\int d^4x\,\sqrt{-g}\frac{\delta V}{\delta\Phi}\delta\Phi = 0\,, \tag{4.14}$$

since we evaluate $\delta V/\delta\Phi$ on the background. Therefore, there are no couplings of the form Φh^N. Similar couplings are allowed, however, when we include higher derivative corrections to the gravitational action, since we cannot rule out terms of the form $\Phi(R_{\mu\nu\lambda\rho})^N$ for $N \geq 2$ (we can rule out $N = 1$ by converting to Einstein frame for S_{EH}).[6] The simplest trilinear vertex is schematically

$$\mathcal{L}_{\Phi h^2} \approx \lambda\Phi\partial^4 h^2\,, \quad \lambda \lesssim \frac{1}{m_\Phi M_p^2}\,, \tag{4.15}$$

where the limit on the coupling λ is saturated when the mass of the decaying particle is the same as the cutoff of the effective field theory. It seems possible that this coupling is further suppressed by powers of the warp factor w, but we will take (4.15) as an upper bound.

4.2 Brane interactions

Now we turn to trilinear couplings of KK modes with brane fields.

[6] Interestingly, time-varying backgrounds, such as quintessence, or other Lorentz breaking backgrounds can induce interactions as well. However, these are unimportant for us because such interactions are suppressed by the Hubble scale and are further not present for massive fields like KK modes.

4.2.1 D3-brane Standard Model

We first examine models in which the SM lives on D3-branes at the tip of the throat. We consider two types of couplings. First, suppose that the position of the D3-brane on the angular dimensions at the tip of the throat completely breaks the isometries associated with the charge of the lightest charged KK mode. Of course, a single pointlike brane cannot break the entire isometry of the throat, but the KK mode might have motion outside the unbroken isometry group. In practical terms, the angular wave function of $\delta\Gamma^\star$ does not vanish at the position of a D3-brane that breaks the appropriate isometries.

There is first a dimension-5 coupling between $\delta\Gamma^\star$ and the brane scalars, which follows from the scaling of the angular components $\tilde{g}_{mn}$ under fluctuations of the $T^{1,1}$ breathing mode. This is

$$\mathcal{L}_{\gamma^\star\phi^2} \approx \frac{\mu_3}{g_s}\frac{w^2}{k^2}\delta\Gamma^\star(x,y_0)\partial_\mu\phi\partial^\mu\phi \approx k^3\frac{p_1\cdot p_2}{M_s^4 w}\gamma^\star\phi^2\,, \tag{4.16}$$

where we have converted to canonically normalized scalars using (3.7) after the second approximate equality and used the fact that $e^{(2+\nu^\star)kz_0} = w^{-(2+\nu_\star)}$ for a brane at the tip of the throat. We have assumed that the angular part of the $\delta\Gamma^\star$ wave function is of order unity. As in the axion interactions of section 4.1.3, $p_{1,2}$ are the momenta of the brane scalars. For $k \lesssim M_s$, this is suppressed by the warped string scale wM_s.

Similarly, there is a Yukawa coupling with the brane fermions. From [49] (see also [50, 51]), the fermionic action on a D3-brane includes the terms

$$\mathcal{L}_{\Theta_3} \approx g_s w\,\bar{\Theta}_3\Gamma^{mnp}\Theta_3\,\mathrm{Re}\,(iG-\tilde{\star}_6 G)_{mnp}\,, \tag{4.17}$$

where Γ^m is the Dirac matrix associated with the full compact metric including warp factors, and Γ^{mnp} is the antisymmetrized product of $\Gamma^m\Gamma^n\Gamma^p$. We have normalized the fermions canonically. The flux component in (4.17) is

$$\mathrm{Re}\,(iG-\tilde{\star}_6 G)_{z\theta\phi} = g_s^{-1}\delta\Gamma^\star H_{z\theta\phi}\,,\quad \mathrm{Re}\,(iG-\tilde{\star}_6 G)_{\theta\phi\psi} = -g_s^{-1}\delta\Gamma^\star H_{\theta\phi\psi}\,. \tag{4.18}$$

In the usual KS solution, the NSNS flux H has a leg on z, but the parametrics of the interaction are the same for either component, and some 4D fermions couple to $\delta\Gamma^\star$ in either case. Then

$$\mathcal{L}_{\gamma^\star\bar{\Theta}_3\Theta_3} \approx wk\mathrm{Re}(iG)\delta\Gamma^\star(x,y_0)\bar{\Theta}_3\Theta_3 \approx \left(\frac{k}{M_s}\right)^4\gamma^\star\bar{\Theta}_3\Theta_3 \tag{4.19}$$

with canonically normalized $\gamma^\star(x)$.

The second class of couplings is the one obtained by substituting $\delta\Gamma$ for $\delta\Gamma^\star$. In fact, the scaling and dependence on all variables is the same as in (4.16) and (4.19). These couplings are important in the case that $\delta\Gamma^\star$ vanishes at the position of the D3-brane, either because the brane does not break the appropriate isometry (including the case that the $\delta\Gamma^\star$ KK mode has a vanishing wave function at z_0 due to a centrifugal barrier). It should be noted that the volume modulus has a suppressed coupling to D3-branes due to the small warp factor at the tip, so we do not consider this coupling.

4.2.2 D7-brane Standard Model

We now turn to models with the Standard Model living on D7-branes in the KS throat. Similarly to the D3-brane Standard Model, we are interested in two sets of interactions, those directly coupling $\delta\Gamma^\star$ to the D7-brane fields and those coupling $\delta\Gamma$ to the brane fields. A point of interest, of course, is whether the D7-brane preserves enough isometries to forbid some of the direct couplings. While a particular D7-brane embedding realizes only one of these possibilities, we consider both cases since more general embeddings probe them both.

In either case, the interaction arises once again because the DBI action of the brane is given by the induced metric on the brane. Including the $T^{1,1}$ breathing mode in the metric (3.9), the breathing mode should appear in $\tilde{g}_{\chi\bar{\chi}}$ as well as in the longitudinal angular components (some of the y^i directions) within the KS throat. This is simply because χ is approximately an angular coordinate.

Suppose first that the D7-brane breaks enough isometries that the world-volume angular integral of $\delta\Gamma^\star$ does not vanish. In this case, the vertex is given by the integral (3.10) with $\delta\Gamma^\star$ inserted and canonical normalization taken for all fields. This is approximately

$$\mathcal{L}_{\gamma^\star\chi^2} \approx \left(\frac{M_s}{M_p}\right)^{4/3} \frac{w}{k} \left(\frac{w}{w_1}\right)^{\nu^\star} p_1\cdot p_2\, \gamma^\star |\chi|^2 \, . \tag{4.20}$$

Even in the case that the D7-brane maintains the isometries, there can be a direct coupling of $\delta\Gamma^\star$ to the brane along with the indirect coupling through a $\delta\Gamma$ intermediate state. Both of these are summarized in the term

$$\mathcal{L}_\chi \approx \int_0^{z_1} dz \int d^3\theta \sqrt{\hat{g}} e^{-2kz} \left(\delta\Gamma + \Delta\Gamma\delta\Gamma^\star\right) \partial_\mu\chi\partial^\mu\bar{\chi} \, , \tag{4.21}$$

where we have again used canonically normalized χ. These couplings integrate to

$$\mathcal{L}_{\gamma\chi^2} \approx \left(\frac{M_s}{M_p}\right)^{4/3} \frac{w}{k} \left(\frac{w}{w_1}\right)^{\nu} p_1\cdot p_2\, \gamma|\chi|^2 \, , \quad \mathcal{L}_{\gamma^\star\chi^2} \approx \left(\frac{M_s}{M_p}\right)^{4/3} \frac{w^3}{k} \left(\frac{w}{w_1}\right)^{2+\nu^\star} p_1\cdot p_2\, \gamma^\star|\chi|^2 \tag{4.22}$$

with canonical normalization. Moreover, the universal volume modulus couples similarly to the brane scalars; in this case, we can just read the coupling from (3.10) through the shift of the warp factor (3.4). If we approximate V_{4w} as below eq. (3.10) which is reasonable, since we work in coordinates where the warp factor is nearly unity in the bulk CY, we immediately find

$$\mathcal{L}_{c\chi^2} \approx \frac{p_1\cdot p_2}{M_p} c|\chi|^2 \, . \tag{4.23}$$

We would also like to consider Yukawa interactions with the D7-brane fermions. Unfortunately, the full couplings of D7-brane fermions to supergravity modes are not known in the presence of 3-form flux and general warp factors.[7] In principle, the Yukawa coupling can be determined by expanding the action given in [58], but that is beyond the scope of this paper. However, 3-form flux can induce Yukawa interactions [50], and we can estimate

[7] In particular, note that [50] specifically assumes that the warp factor is constant parallel to the brane; this choice also obscures where the warp factor would appear implicitly.

the Yukawa couplings simply, as follows. To order of magnitude, the D7-brane fermions and scalars should couple similarly to the closed string degrees of freedom, but the dimension-5 operators should be suppressed by a single power of the cutoff of the D7-brane effective field theory. This scale is approximately $w_1 k$, so we can estimate the corresponding Yukawa couplings by multiplying, for example, (4.22) by $w_1 k$ (and removing the $p_1 \cdot p_2$ factor):

$$\mathcal{L}_{\gamma\bar{\Theta}_7\Theta_7} \approx \left(\frac{M_s}{M_p}\right)^{4/3} w^2 \left(\frac{w}{w_1}\right)^{-1+\nu} \gamma\bar{\Theta}_7\Theta_7 \,, \quad \mathcal{L}_{\gamma^\star\bar{\Theta}_7\Theta_7} \approx \left(\frac{M_s}{M_p}\right)^{4/3} w^4 \left(\frac{w}{w_1}\right)^{1+\nu^\star} \gamma^\star\bar{\Theta}_7\Theta_7 \,. \tag{4.24}$$

Note that the D3-brane couplings satisfy this relation as well, with cutoff wk.

5 Decay rates

In this section, we assemble the interactions into decay rates for the DM candidates. There are several different possible rates, depending on specifics of the Standard Model construction, the CY manifold, and the specific DM candidate.

When making numerical estimates for the decay rates, we use the following parameter values appropriate for TeV scale dark matter, as indicated in the decay rates below: $M_s \approx M_{GUT} \approx 10^{16}$ GeV, $k \approx M_s$, $w \approx 10^{-13}$, and $w_1 \approx 10^{-4}$. The value of w_1 assumes an intermediate scale for the D7-brane SM with the SM hierarchy generated by supersymmetry breaking (this is the same scale as discussed in [52], for example).

5.1 Decays to supergravity modes

We first consider decays to light supergravity modes, specifically moduli and their axion partners. After moduli stabilization, we expect that these are actually heavy compared to the lightest Standard Model particles; assuming reasonable couplings, these supergravity modes can decay quickly to SM states. On the other hand, if the light supergravity states couple only weakly to the SM, they may constitute a different candidate DM sector, along the lines suggested in ref. [59]. We leave this possibility for other work, as we are interested in the lifetime of the charged KK mode sector as DM candidates.

It turns out that both dimension-3 and dimension-5 interactions are important for decays to light supergravity modes, so we consider both here. We begin with decays to the universal volume modulus, which should be representative of decays to other uncharged moduli. This decay can proceed through two channels: first, our DM candidate $\delta\Gamma^\star$ has a direct dimension-3 coupling (4.8) with two moduli; second, $\delta\Gamma^\star$ can oscillate into a volume modulus through (4.7), which then decays via the dimension-5 self-coupling (4.9). The amplitudes for both of these processes are parametrically similar, and they yield a decay rate of order (using $\Gamma \sim m_\star^{-1}|\mathcal{M}|^2$ for a scalar of mass $m_\star$ decaying into much lighter particles)

$$\begin{aligned}\Gamma(\gamma^\star \to cc) &\approx \frac{M_s^8}{M_p^4 k^3} w^{9+2\nu^\star} \\ &\approx 10^{-113-26\nu^\star}\,\text{GeV} \times f(-3,5,9+2\nu^\star) \\ &\approx 10^{-89-26\nu^\star}\,\text{s}^{-1} \times f(-3,5,9+2\nu^\star) \,.\end{aligned} \tag{5.1}$$

where

$$f(i,j,k) = \left(\frac{k}{M_s}\right)^i \left(\frac{M_s}{10^{16}\ \text{GeV}}\right)^j (10^{13}w)^k\ . \tag{5.2}$$

Although the two diagrams do not cancel to the approximations we have used, we note that they could possibly cancel in a more complete calculation, even though there is no apparent symmetry-based reason that they should. In that case, the decay proceeds through a dimension-5 coupling, which, as we now show, may be competetive with (5.1) in some circumstances.

We estimate these decay rates by considering the dimension-5 couplings to the universal axion given in (4.10), (4.11). The amplitudes involving these vertices (and quadratic mixing when appropriate) have different powers in the warp factor, so one or the other process dominates in different models. For $\nu < \nu^\star + 2$, the process with γ-$\gamma^\star$ mixing (4.6), intermediate γ state and vertex (4.11) dominates and yields a decay rate which is equal to that which we would get from $\gamma \to 2a$, but suppressed by the mixing angle squared $\theta^2 \sim w^4$. The result is

$$\Gamma(\gamma^\star \to aa) \ = \ \Gamma(\gamma^\star \to cc) \text{ with } \nu^\star \to \nu \tag{5.3}$$

which indeed is larger than (5.1) when $\nu < \nu^\star$. (In the intermediate state propagator, we have assumed that the mass squared of both $\delta\Gamma^\star$ and $\delta\Gamma$ are of order wk, as is the difference, and we continue to make this assumption throughout; furthermore, although we should sum over the entire radial KK tower, the sum is convergent and should not affect order of magnitude estimates.) For $\nu > \nu^\star + 2$, the direct $\gamma^\star \to 2a$ decay amplitude (4.12) dominates, and we find

$$\begin{aligned}\Gamma(\gamma^\star \to aa) &\approx \frac{M_s^8}{M_p^4 k^3} w^{13+2\nu^\star} \\ &\approx 10^{-165-26\nu^\star}\text{GeV} \times f(-3,5,13+2\nu^\star) \\ &\approx 10^{-141-26\nu^\star}\text{s}^{-1} \times f(-3,5,13+2\nu^\star)\ .\end{aligned} \tag{5.4}$$

Recall that the charged KK state can decay to an axion with angular charge in the throat and an uncharged axion (such as the universal axion) through the direct dimension-5 coupling (4.13), assuming such a charged modulus and axion exist. In this case, we find a much faster decay channel,

$$\begin{aligned}\Gamma(\gamma^\star \to aa^\star) &\approx \frac{M_s^8}{M_p^4 k^3} w^{5+2\nu^\star} \\ &\approx 10^{-61-26\nu^\star}\text{GeV} \times f(-3,5,5+2\nu^\star) \\ &\approx 10^{-37-26\nu^\star}\text{s}^{-1} \times f(-3,5,5+2\nu^\star)\ .\end{aligned} \tag{5.5}$$

Despite the fact that this is by far the fastest of the decay rates we have found to the modulus/axion sector, the lifetime is still many orders of magnitude longer than the age of the universe.

Finally, we can give an upper bound for decay to 4D gravitons using the inequality in eq. (4.15) for the dimension 7 coupling λ. Taking $m_\Phi \sim wk$ and also replacing the graviton derivatives by $\partial \to wk$, we find a decay rate

$$\begin{aligned}\Gamma(\gamma^\star \to hh) \lesssim \frac{w^5k^5}{M_p^4} &\approx 10^{-61}\ \text{GeV}\ \times\ f(5,5,5)\\ &\approx 10^{-37}\ \text{s}^{-1}\ \times f(5,5,5)\ .\end{aligned} \tag{5.6}$$

This is much faster than the other decays to bulk supergravity modes (except for (5.5) when $\nu^\star \sim 0$), but it still results in a lifetime which is much much longer than the age of the universe. Notice that this is an upper bound on the rate of decay into gravitons.

5.2 Decays to D3-brane Standard Model

Now we turn to decays to SM particles with the SM living on D3-branes at z_0. There are two possibilities for these decays: either $\delta\Gamma^\star$ has a vanishing wave function at the position of the brane (for symmetry reasons such as a centrifugal barrier) or not. We begin with the case in which there is a nonvanishing direct coupling of $\delta\Gamma^\star$ to the brane.

5.2.1 Direct decay through symmetry breaking

In this case, the direct interactions of $\delta\Gamma^\star$ with the brane fields are given by (4.16) and (4.19) for couplings to the brane scalars and fermions respectively. For scalars, the rate simply follows from the usual formula $\Gamma \sim m_\star^{-1}|\mathcal{M}|^2$, where the Feynman diagram for the decay is just the vertex. Recall that the inner product of momenta is $p_1 \cdot p_2 \sim wk$ for decay products much lighter than the DM candidates. Then the decay rate is

$$\Gamma(\gamma^\star \to \phi\phi)\ \approx\ \frac{wk^9}{M_s^8}\ \approx\ 1\ \text{TeV} \times f(9,1,1)\ \approx\ 10^{27}\ \text{s}^{-1} \times f(9,1,1)\ . \tag{5.7}$$

This is extremely rapid (TeV scale) and would obviously rule out these angular KK modes as DM candidates if it is not forbidden.

The decay rate to fermions also follows from a single vertex; the only additional element is the trace over external spinors, which simply gives $\text{tr}[\not{p}_1\not{p}_2] \sim p_1 \cdot p_2$. Working through the algebra, we find the same decay rate (5.7) to order of magnitude even though the scalar decay proceeded via a dimension-5 operator; that is, $\Gamma(\gamma^\star \to \bar{\Theta}_3\Theta_3) \approx \Gamma(\gamma^\star \to \phi\phi)$. In retrospect this is understandable because the DM mass is similar to the natural cutoff of the D3-brane theory.

It is noteworthy that there exists a simple dynamical mechanism whereby these fast decay channels can be forbidden. Namely, the breaking of angular isometries implies that the brane modulus is stabilized on some particular subspace of the $T^{1,1}$ manifold. It is plausible that the potential is minimized for angles where wave function $Y(\theta_i)$ vanishes. As a specific example, [22, 41] showed that states charged under one of the SU(2) factors face a centrifugal barrier due to the shrinking of the S^2 at the tip of the throat, so the wave function vanishes at the tip. This is related to the fact that this SU(2) factor rotates the $\mathbb{R}^3$ formed by the shrinking S^2 and the radial direction, a symmetry unbroken by the D3-brane at the tip, the origin of that $\mathbb{R}^3$. Then the brane position does not break the symmetry generator under which the KK mode has charge.

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5.2.2 Indirect decay only

If there is a dynamical mechanism such as the one mentioned above, then the lightest charged state has a vanishing wave function on the D3-brane. Alternatively, it could happen that the DM candidate has angular momentum on the 2-cycle that shrinks at the throat tip; this is in fact a variation of the previous option, as the D3-brane position on the S^3 at the tip does not break angular symmetries of the (shrinking) S^2.

In such cases the decay amplitude includes mixing of $\gamma^\star$ with γ, a γ propagator, and the vertex of γ on the brane. As in the case of the direct decay, the decay rates into scalars and fermions are parametrically similar, and the common rate is

$$\begin{aligned}\Gamma(\gamma^\star \to \phi\phi) \approx \Gamma(\gamma^\star \to \bar{\Theta}_3\Theta_3) &\approx \frac{w^5 k^9}{M_s^8} \\ &\approx 10^{-49}\ \mathrm{GeV} \times f(9,1,5) \approx 10^{-25}\ \mathrm{s}^{-1} \times f(9,1,5)\ . \end{aligned} \tag{5.8}$$

This rate is particularly interesting; for the fiducial values we have chosen, it is close to that needed to explain the high-energy positron excess seen by PAMELA and electron-positron flux excess seen by Fermi, ATIC, HESS, and other experiments [60, 61]. Recall that in a realistic embedding of the SM on the brane, Θ_3 would represent SM fermions, gaugini, or Higgsino, and ϕ would be the Higgs boson or sfermions of the MSSM. We have dropped some model-dependent factors such as flux quantum numbers which can shift this prediction by a few orders of magnitude, but one immediately sees from (5.8) that slight changes in the AdS_5 radius, warp factor, or string scale can compensate for that. One interesting direction for future research would be to determine if this class of models can be arranged to provide leptophilic DM decays, which would be consistent with observation. However, given the difficulty of that task, it is perhaps more interesting and important to use this rate to constrain D3-brane Standard Models. The point is that any DM decay to SM particles of this rate or above would be detected; therefore, if a detailed thermal history yields an appropriate density of charged KK states, certain throat geometry and flux configurations can be ruled out. Specifically, it may be possible to rule out some region of w, k, M_s parameter space.

5.3 Decays to D7-brane Standard Model

The last category of decays we consider is to SM particles with the SM on a D7-brane which extends some distance into the KS throat. Again, there are the same two cases as for the D3-brane Standard Model.

5.3.1 Direct decay through symmetry breaking

In this case, as before, the position of the D7-brane breaks the isometry associated with the angular motion of the KK DM candidate. Then the world-volume integral of $\delta\Gamma^\star$ does not vanish, so $\gamma^\star$ can decay by the interaction (4.20) to D7-brane scalars. Using that interaction, the rate comes to

$$\begin{aligned}\Gamma(\gamma^\star \to \bar{\chi}\chi) &\approx w^5 k \left(\frac{M_s}{M_p}\right)^{8/3} \left(\frac{w}{w_1}\right)^{2\nu^\star} \\ &\approx 10^{-57-18\nu^\star}\ \text{GeV} \times f(1, 11/3, 5)\, g(2\nu^\star) \\ &\approx 10^{-33-18\nu^\star}\ \text{s}^{-1} \times f(1, 11/3, 5)\, g(2\nu^\star)\ . \end{aligned} \tag{5.9}$$

where $g(n) = \left(10^9 w/w_1\right)^n$. For small values $\nu^\star \gtrsim 0$ and large w ($w \gtrsim 10^{-12}$), this decay mode comes close to being observable. With care, some corner of parameter space could be ruled out for these models; this statement is stronger for SM fermions, as we now see.

We can also estimate the decay to SM fermions in this case. Using the interaction (4.24) leads to the same rate as (5.9) except rescaled by $(w_1/w)^2$. Thus, we estimate the decay to D7-brane fermions as

$$\begin{aligned}\Gamma(\gamma^\star \to \bar{\Theta}_7\Theta_7) &\approx w^5 k \left(\frac{M_s}{M_p}\right)^{8/3} \left(\frac{w}{w_1}\right)^{-2+2\nu^\star} \\ &\approx 10^{-39-18\nu^\star}\ \text{GeV} \times f(1, 11/3, 5)\, g(-2+2\nu^\star) \\ &\approx 10^{-15-18\nu^\star}\ \text{s}^{-1} \times f(1, 11/3, 5)\, g(-2+2\nu^\star)\ . \end{aligned} \tag{5.10}$$

Again, this rate can become observable in the large w region of parameter space for $\nu^\star \lesssim 1$. This model is a good candidate for future detailed study, including a careful calculation of the Yukawa coupling.

Notice that these decay rates are slower than for the corresponding case of decay to a D3-brane SM. The reason is that the wave function of the KK mode DM is localized near the tip of the throat and has maximal overlap with the D3-brane, while the D7-brane extends only a relatively short distance into the throat where the KK mode wave function is small.

5.3.2 Indirect decay only

If the D7-brane does not break the appropriate symmetries of the lightest charged KK mode, then the decay amplitude involves the throat deformation $\Delta\Gamma$ and, in some terms, intermediate states. There are three relevant diagrams: one with $\delta\Gamma^\star$ scattering off $\Delta\Gamma$ on the brane, one with a $\delta\Gamma$ intermediate state, and one with the universal volume modulus as the intermediate state. These three diagrams give parametrically different contributions to the decay amplitude, so they dominate in different models.

The diagram with the universal volume modulus (or other uncharged modulus) as an intermediate state dominates the decay amplitude when $\nu^\star$ is particularly small or when the D7-brane does not stretch very far into the throat (w_1 is large). In fact, this process can occur even if the D7-brane is localized entirely in the bulk CY, due to the fact that the modulus wave function spreads throughout the entire compactification. Putting together the quadratic mixing (4.7), the propagator discussed in section 5.1 above, and the interaction (4.23), we find approximately

$$\begin{aligned}\Gamma(\gamma^\star \to \bar{\chi}\chi) &\approx w^{9+2\nu^\star}\frac{M_s^8}{M_p^4 k^3} \\ &\approx 10^{-113-26\nu^\star}\ \mathrm{GeV} \times f(-3,5,9+2\nu^\star) \\ &\approx 10^{-89-26\nu^\star}\ \mathrm{s}^{-1} \times f(-3,5,9+2\nu^\star)\ . \end{aligned} \tag{5.11}$$

For longer throats with $\nu^\star < \nu + 2$, the dimension-5 coupling of $\delta\Gamma^\star$ to χ dominates. In that case, the decay rate is

$$\begin{aligned}\Gamma(\gamma^\star \to \bar{\chi}\chi) &\approx kw^9\left(\frac{w}{w_1}\right)^{4+2\nu^\star}\left(\frac{M_s}{M_p}\right)^{8/3} \\ &\approx 10^{-145-18\nu^\star}\ \mathrm{GeV} \times f(1,11/3,9)\, g(4+2\nu^\star) \\ &\approx 10^{-121-18\nu^\star}\ \mathrm{s}^{-1} \times f(1,11/3,9)\, g(4+2\nu^\star)\ . \end{aligned} \tag{5.12}$$

In other cases, the diagram with an intermediate uncharged KK mode dominates the decay amplitude. Then the decay rate is

$$\begin{aligned}\Gamma(\gamma^\star \to \bar{\chi}\chi) &\approx kw^9\left(\frac{w}{w_1}\right)^{2\nu}\left(\frac{M_s}{M_p}\right)^{8/3} \\ &\approx 10^{-109-18\nu}\ \mathrm{GeV}\ \times f(1,11/3,9)\, g(2\nu) \\ &\approx 10^{-85-18\nu}\ \mathrm{s}^{-1} \times f(1,11/3,9)\, g(2\nu)\ . \end{aligned} \tag{5.13}$$

Notice that all the implied lifetimes are much longer than the age of the universe.

To find decay rates to D7-brane fermions, our estimates tell us to multiply (5.11), (5.12), (5.13) by $(w_1/w)^2$. For the fiducial values we have chosen, this is a factor of 10^{18}, which still leaves all the indirect decay rates much slower than the current Hubble rate. In this estimate, the decay channel with the intermediate modulus now depends on w_1; when the D7-brane does not extend into the KS throat containing the DM candidate, set $w_1 \to 1$. However, $\gamma^\star$ and γ cannot couple to D7-branes that do not extend into the throat.

5.4 Tunneling to other throats

In the case that the SM branes are located outside the throat, our DM candidates may decay to supergravity modes as discussed in 5.1 above. Alternatively, though, there are possible decay channels to the SM sector, even in other throats. One possibility, which we have already mentioned in the caes of a D7-brane SM, is that the KK mode mixes with a modulus (the universal volume modulus, for example), which couples to branes throughout the compactification, albeit weakly. This process is related to the discussion of [23, 27] for uncharged KK modes coupling to gravitons.

Another possibility is that KK modes may tunnel through the bulk CY to couple to the SM degrees of freedom. Similar tunneling rates have been discussed in the context of reheating in [22, 23, 41, 62–64] (and more generally in [65, 66]); there is some support for the idea that the longest throats reheat the most. This gives a reason to consider the SM to live in the throat with our DM candidates, but we consider other models to be comprehensive.

Tunneling is closely tied to the cosmological history of dark matter, including reheating and thermalization, but we give a brief analysis of its importance in DM decays here.

We will adopt the estimate of [23] for the tunneling amplitude, which is given by a mixing angle $\sim w_A^4$ between uncharged KK modes localized near the tip of throat A and modes localized in a second throat B.[8] Note that this amplitude is independent of the length of the throat B containing the final state particles. However, there is as yet no analysis of tunneling rates for states charged under approximate isometries of throat A; since the final state is in a different throat with different (or no) approximate isometries, the tunneling rate may be affected. Without performing a careful calculation, we consider two extremes: first, that the tunneling amplitude is unaffected due to the fact that the bulk, which the particle tunnels through, breaks all isometries strongly, and, second, that the charged KK mode must oscillate into an uncharged KK mode before tunneling. The latter eventuality suppresses the tunneling amplitude by an additional factor of w_A^2 (see (4.6) and the subsequent discussion).

There are two major cases for the final states after tunneling. If the KK spectra of the two throats match closely, tunneling can occur nearly on-shell. In that case, the KK mode of throat A oscillates into a KK mode of throat B, which later decays to the SM. The two extreme possibilities for this tunneling rate are

$$\begin{aligned}\Gamma(\gamma_A^\star \to \mathrm{KK}_B) &\approx w_A^9 k_A \approx 10^{-101}\ \mathrm{GeV}\ \times f(1,1,9)\\ &\approx 10^{-77}\ \mathrm{s}^{-1}\ \times f(1,1,9) \quad \textbf{or}\\ \Gamma(\gamma_A^\star \to \mathrm{KK}_B) &\approx w_A^{13} k_A \approx 10^{-153}\ \mathrm{GeV}\ \times f(1,1,13)\\ &\approx 10^{-129}\ \mathrm{s}^{-1}\ \times f(1,1,13)\ .\end{aligned} \tag{5.14}$$

Note, though, that this does not give the direct decay rate into light particles; the KK_B lifetime could still be significant compared to particle physics scales or even the age of the universe, though we assume that it is short compared to the tunneling rate.

If the KK spectra of the two throats are very different, then the B-throat KK mode must be treated as an off-shell intermediate state. In this case, the decay amplitude takes the form

$$\text{tunneling mixing angle} \times\ \mathrm{KK}_B \text{ decay amplitude}\,, \tag{5.15}$$

perhaps with oscillation into an uncharged KK mode of throat A before tunneling. As an example, consider direct decay to a D3-brane at the tip of throat B. In that case, we find

$$\Gamma(\gamma_A^\star \to \phi_B\phi_B) \approx w_A^8 \frac{w_B k_B^9}{M_s^8} \quad \text{or} \quad w_A^{12} \frac{w_B k_B^9}{M_s^8}\ . \tag{5.16}$$

These rates clearly depend on the parameters of the B throat.

[8] Note that this is more suppressed than earlier estimates based on [65, 66]; we favor the results of [23] because that work glues the throat to the bulk CY in a smooth fashion, more representative of the full compactification. However, as we indicate, tunneling in warped compactifications is not completely understood and is worth revisiting.

5.5 Decay rate generalizations

We conclude our presentation of decay rates with a few general comments.

First, we note that we have already, in a subtle way, presented the decay rates of uncharged KK modes. Any decay of $\gamma^\star$ that does not require an insertion of $\Delta\Gamma$ corresponds to a similar decay of γ. For example, (5.5) gives a decay rate for $\gamma^\star \to aa^\star$, but the decay $\gamma \to aa$ is parametrically identical. Similarly, the couplings of $\gamma^\star$ to isometry-breaking branes are identical to the couplings of γ to any brane (taking $\nu^\star \to \nu$ when appropriate). Therefore, we can immediately see that an uncharged KK mode in a TeV scale throat with a D3-brane at the tip will decay with a TeV scale rate.

We can also give some simple conversion rules for our decay rates in the case that the lightest charged KK state carries a different angular charge. As mentioned in section 4.1.1, the small perturbation $\Delta\tilde{\Gamma}$ with charge $(1/2, 1/2, 1)$ grows as $e^{5kz/2}$ in the throat. If the lightest charged KK mode falls in this charge sector, then decay amplitudes involving an insertion of $\Delta\tilde{\Gamma}$ are boosted by a factor w^{-1}, assuming that the integrals done in the dimensional reduction are dominated by large z. These include the $\gamma^\star$–γ mixing (4.6), the $\gamma^\star$–c mixing (4.7) for $\nu^\star > 7/2$, the $\gamma^\star$–a–$a^\star$ coupling (4.13) for $\nu^\star > 7/2$ (or other $\gamma^\star$-axion couplings at large $\nu^\star$), and trilinear $\gamma^\star$-D7-brane couplings (4.22), (4.24). On the other hand, this deformation can be forbidden by certain discrete symmetries of the compact CY, but those symmetries would only shift the mass spectrum of $(1/2, 1/2, 1)$ KK modes, not project them out entirely. If those KK modes are the lightest charged states (which may or may not be the case), the DM decays will be suppressed due to requiring multiple insertions of other deformations (absent isometry-breaking D-branes, that is).

There are, of course, other isometry-breaking deformations, most of which correspond to irrelevant operators in the gauge theory and therefore decay exponentially in z. If the lightest KK state decays through a process with a single deformation inserted, the analysis will be very similar to that presented here. However, due to the difference in z dependence and coefficients, the parametrics will differ from our results. Some of these deformations and decays, including amplitudes involving multiple deformation insertions, were discussed in [25].

Finally, the decays of KK modes in a throat that supports brane inflation are important in the process of reheating. These are the same processes that we have discussed, but the throat is typically much shorter, $w \sim 10^{-3}$ or 10^{-4}.

6 Discussion

In this section we give a brief comparison to previous studies of KK modes in warped throats of string compactifications, then we outline some interesting future directions for the subject. But first let us quickly recap our results. Most of the decay channels we have considered, which should be representative of DM decays into the given sectors, result in decay rates which are slow compared to the age of the universe. Therefore, angularly charged KK modes are suitable dark matter candidates in compactifications in which those decays dominate. However, if the Standard Model is supported on D-branes with extent in the same warped throat that the KK modes occupy, the decay rates may be much faster.

For example, the natural decay rate onto a D3-brane that breaks the relevant angular isometries is TeV scale, just set by the warped scale at the bottom of the throat, which is also the mass scale of the KK mode.

On the other hand, having the SM supported on a D3-brane that respects the isometries of the DM candidate leads to an estimated decay rate very similar to the observed bounds. In particular, a careful enough study may be able to constrain the parameter space of D3-brane SM constructions in a class of compactifications. A more exciting possibility is that hints for decaying dark matter might be confirmed in currently running experiments such as Fermi-LAT. We can make a similar statement for D7-brane SM constructions that break the appropriate isometries of the throat, though the relevant parameter space has an extra dimension and the decay rates are more sensitive to discrete compactification choices (represented by ν and $\nu^\star$).

6.1 Comparison to previous results

As we discussed in the introduction, several groups have considered KK modes as DM candidates in string compactifications before. Proceeding thematically rather than chronologically, [23, 27] have considered KK modes of separate warped throats as DM candidates. That is, they consider compactifications with multiple throats and take KK modes (without angular charge) as DM candidates. They also considered decays to the SM mediated by supergravity modes, which are similar in spirit to the D7-brane SM decays mediated by the volume modulus in section 5.3. In addition, [27] use a gauge theory dual to KS throats, which is appropriate in a different regime of validity than the supergravity description of the throats. In terms of the estimated decay rates, however, the biggest difference with our results is that [27] does not account for the approximate isometry of the throats, which may lead them to overestimate some of the decay rates by overestimating the mixing with light supergravity modes.

Next, [23] and [24] have, between them, given a detailed cosmological history of KK modes, including charged KK modes as DM candidates, but in a truncated theory with only KK modes of the 4D graviton. Among decay channels, [24] considered decays to a D3-brane SM sector; unsurprisingly, since the graviton couples to the brane scalar kinetic term like our KK modes, they found similar decay rates to D3-brane degrees of freedom. On the other hand, graviton KK mode interactions are somewhat constrained by orthogonality relations, so we might expect correspondingly richer physics in the thermal history of the full supergravity KK spectrum.

Our work is most closely related to that of [25]. That paper also considered the full IIB supergravity and attempted to identify the lightest charged state. However, it did not account for contributions to the mass beyond the geometrical contribution from $T^{1,1}$ dimensional reduction. Furthermore, it did not allow relevant (growing) deformations of the KS throat, which removed the $(1,0,0)$ $\Delta\Gamma$ deformation from consideration. Therefore ref. [25] did not consider as large a range of decay channels as we have examined. Lastly, [25] was mostly concerned with KK modes of a throat that supports brane inflation in order to determine whether charged KK modes yield dangerous relics which could overclose the universe. A decay rate to gravitons was found to be

$$\Gamma(\gamma^\star \to hh) \approx w^{3.4} \frac{M_{3/2}^2 k}{M_p^2}\,, \tag{6.1}$$

where $M_{3/2}$ is the amplitude of a supersymmetry-breaking deformation at the bulk ($z = 0$), which [25] approximated as parametrically similar to the supersymmetry breaking scale (*ie.*, gravitino mass). $M_{3/2}$ therefore encodes the information about the strength of isometry breaking carried by this particular deformation. Following [67], generically the supersymmetry-breaking scale satisfies $M_{3/2}^2 M_p^2 \approx w^4 M_s^4$. Given that we are now considering an inflationary throat, larger values of $M_{3/2}$ tend to destabilize the inflationary potential, leading to runaway directions rather than slow-roll inflation. Then this decay rate becomes (at most)

$$\Gamma(\gamma^\star \to hh) \approx w^{7.4} \frac{M_s^4}{M_p^4} k\,. \tag{6.2}$$

This rate is matched or exceeded by a number of the decay channels we have considered, including all decays to SM modes on D3-branes, decays to axions ($\gamma^\star \to aa^\star$), possibly gravitons ($\gamma^\star \to hh$, using our new estimate) and likely the direct decays to SM modes on D7-branes. It is possible that both this estimate of $M_{3/2}$ and the amplitude of the $\Delta\Gamma$ deformation we study can vary in multi-throat scenarios, although where the current estimates might break down is not entirely clear at this time. However, we can still attempt to address this uncertainty by considering a throat for the SM, as in the rest of this paper, and considering TeV-scale supersymmetry breaking, $M_{3/2} \sim wk$. Even in that case, the decay (6.1) is still suppressed compared to several of our decay rates.

Most recently, [28] constructed a model designed to yield decay rates of the dark matter into SM particles at just the rate needed to explain the astrophysical electron and positron excesses seen by PAMELA, Fermi, ATIC, HESS, and other experiments [60, 61]. In that model, the dark matter candidate is a KK mode of D7-brane fields extending to the tip of a TeV scale throat. The KK mode decays first to a light messenger, which then decays to SM modes. We have taken a different approach; rather than design a model to give observable decay rates, we have instead surveyed generic configurations in the class of conformally CY warped compactifications of type IIB string theory for possible DM candidates. We find that several of these backgrounds support DM candidates with lifetimes of order 10^{25} s, as desired to match the observed anomalies, although our decay products are not necessarily limited to electrons and positrons. As we mentioned in section 5.2.2, it is plausible that some models give naturally leptophilic couplings for the dark matter, depending sensitively on the geometry of the embedding of SM branes in the warped throat and the choice of background flux. However, a potentially more interesting question is how strongly similar string compactifications can be constrained in order to avoid decay rates, leptophilic or not, which are already too large compared to observational bounds, especially since such constraints may apply more generically than any specific model to reproduce the known observational anomalies.

6.2 Future directions

We conclude by discussing several interesting directions for future work. First, we have highlighted two sets of couplings of interest to string phenomenology. One is the coupling of general supergravity fermions to brane fermions; see section 3.4. The other, discussed in section 3.3.2, is the coupling of D7-brane fermions to bosonic supergravity fields in the presence of a general warp factor. These couplings are necessary to complete our survey of dark matter candidates, but they are also more generally of interest for the phenomenology of brane constructions. In terms of decay rates for our KK DM candidates, it would also be worthwhile to determine the effects of approximate isometries and their breaking in the bulk CY on tunneling rates. In addition, a deeper understanding of quantum mechanical and α' corrections to the compactification geometry would lead to a more accurate accounting of isometry breaking in these throats. In particular, it would be good to confirm that the estimated strength of isometry breaking is generic, as currently expected.

The obvious next step is to detail the cosmic history of the dark matter candidates we have discussed in this paper, as was done for graviton KK modes in refs. [23, 24]. This would require estimating the couplings between the various charged and uncharged KK modes and the moduli. While we were able to ignore many of the fields of type IIB supergravity in calculating decay rates, thermalization rates are sensitive to interactions among all the fields. This is therefore a more ambitious project. In addition, it may be necessary to understand the full dynamics of long throats during high-scale inflation and reheating, as emphasized in refs. [63, 68]. Some steps have been taken in this direction by refs. [44, 46, 48], although it seems likely that the effects of the inflationary potential will need to be incorporated in the 10D theory, rather than just the 4D effective theory.

Such a careful accounting of cosmological history might provide constraints on the parameter space of D-brane Standard Model constructions with warped hierarchies. Assuming the charged KK modes are populated with the appropriate density, there are both D3-brane and D7-brane models that yield observable (or nearly observable) decay rates in natural regions of parameter space. It is rare to have the opportunity to constrain compactifications of string theory, based on considerations which are independent of the need for embedding the SM; this would give an additional, independent criterion for ruling out regions of the string landscape. Therefore, it would be valuable to study both isometry-preserving D3-branes and isometry-breaking D7-branes to see how strongly their parameter spaces are constrained.

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A D7-brane induced metric for Kuperstein embedding

In this appendix, we demonstrate that the D7-brane scalar kinetic term (3.10) is a good approximation for a D7-brane that obeys the Kuperstein embedding [69]. Such D7-branes are supersymmetric and also have been shown to generate inflationary potentials for probe D3-branes [70, 71]. Recently, [52] studied the excitations of Kuperstein-embedded D7-branes and their couplings to closed string modes; we follow a simplified version of their results.

The metric of the conifold was given as a foliation of Kuperstein embeddings in [52]; this is a convenient set of coordinates to use since we consider fluctuations around a fixed Kuperstein embedding. Therefore, in D7-brane calculations, rather than (2.3), we use

$$d\tilde{s}^2 = \frac{k}{3}e^{kz}\left\{\frac{|\mu+\chi|^2}{2}\left[\frac{1+2\cosh\rho}{3}(d\rho^2+h_3^2)+\cosh^2\left(\frac{\rho}{2}\right)h_1^2+\sinh^2\left(\frac{\rho}{2}\right)h_2^2\right]\right.$$
$$\left.+\frac{2}{3}\sinh\rho(d\rho+ih_3)(\mu+\chi)d\bar{\chi}+\text{c.c.}+\frac{4}{3}(1+\cosh\rho)d\chi d\bar{\chi}\right\} \ . \tag{A.1}$$

(See equation (C.8) of [52] with the supersymmetry breaking parameter $\mathcal{S}=0$.) Our usual radial coordinate z is related to these coordinates by

$$\frac{e^{-3kz}}{k^3} = |\mu+\chi|^2(1+\cosh\rho) \ . \tag{A.2}$$

In the above, the h_i are angular differentials for the angular directions along the brane, and μ is a constant that chooses the location of the embedded D7-brane. We can expand around $\chi=0$, since changing χ just shifts μ (μ acts as the expectation value of χ). In fact, since we are not interested in self-interactions of χ, we can set $\chi=0$ while retaining $d\chi$, and we do so in the following.

A non-fluctuating brane ($\chi=0$) attains its maximum value of $z=z_1$ at $\rho=0$, and we find that

$$\mu^2 = \frac{w_1^3}{2k^3}\ , \quad w_1 \equiv e^{-kz_1} \ . \tag{A.3}$$

Furthermore, just as we approximate the tip of the KS throat with a boundary condition at z_0 for the $AdS_5\times T^{1,1}$ metric, which is valid at $z\ll z_0$, we approximate (A.1) with a metric valid at $z\ll z_1$ and replace the smooth metric near z_1 with a radial boundary condition. Since small z is large ρ, we can replace $\cosh\rho$ and $\sinh\rho$ with $e^\rho/2$ and further $d\rho\sim -3kdz$. Then the $d\chi$ terms in (A.1) become

$$d\tilde{s}^2 \approx \frac{2}{9}\frac{\sqrt{2k}}{w_1^{3/2}}e^{-2kz}\left(-3dz+i\frac{h_3}{k}\right)d\bar{\chi}+\text{c.c.}+\frac{8}{9}\frac{k}{w_1^3}e^{-2kz}d\chi d\bar{\chi}+\cdots\ , \tag{A.4}$$

where we have placed a factor of $1/k$ with the angular differential.

Finally, we need to rescale $\chi,\bar{\chi}$ to match our usual conventions. First, χ as written has dimensions of length$^{3/2}$. To get dimensions of length, we rescale to $\chi'=\sqrt{k}\chi$. The other rescaling we must do is more subtle. The point is that, as $z\to 0$, $\tilde{g}_{mn}$ given in (A.4) should match smoothly to a CY metric $\tilde{g}_{mn}$ with components generically order 1. However, all terms of (A.4) with a factor of $d\chi$ or $d\bar{\chi}$ are larger than order one by a factor of $w_1^{-3/2}$ for each factor of $d\chi$. Therefore, the correct coordinate for matching onto the bulk CY metric is $\chi'' = \chi'/w_1^{3/2}$. (Note that the ρ and angular components of (A.1) are order 1 as $z \to 0$, so those coordinates do not need to be rescaled.)

In the end, we find

$$d\tilde{s}^2 \approx \frac{2\sqrt{2}}{9} e^{-2kz} \left(-3dz + i\frac{h_3}{k} \right) d\bar{\chi}'' + \text{c.c.} + \frac{8}{9} e^{-2kz} d\chi'' d\bar{\chi}'' + \cdots . \tag{A.5}$$

In the full 10D metric, this is multiplied by the warp factor $e^{-2A} = e^{2kz}$. Therefore, the pull-back metric to the D7-brane in static gauge is

$$ds^2 = \left[e^{-2kz}\eta_{\mu\nu} + \frac{8}{9}\partial_{(\mu}\chi\partial_{\nu)}\bar{\chi} \right] dx^\mu dx^\nu + \left[\frac{2\sqrt{2}}{9}\left(-3dz + i\frac{h_3}{k} \right) \partial_\mu \bar{\chi} dx^\mu + \text{c.c.} \right] + \cdots , \tag{A.6}$$

where the $\cdots$ represent terms containing only dz and h_i (and which have no powers of e^{-kz}). The kinetic term, as usual, comes from expanding the determinant of the metric (A.6) to first order in $|\partial\chi|^2$. These terms come from the trace of the fluctuations in the first square brackets of (A.6) (after factoring out the warped Minkowski metric) and also from the square of the off-diagonal terms in the second square brackets (contracted with the warped Minkowski metric). Both of these terms carry a factor of e^{2kz} and are both multiplied by the determinant of the unfluctuated 8D metric, which carries a factor of e^{-4kz}. This yields an overall kinetic term $e^{-2kz}|\partial\chi|^2$ to be integrated over the compact dimensions of the D7-brane, in agreement with (3.10) in the throat.

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